



WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

IMPROVING THE INFORMATIONAL CONTENT OF THE SIMPLE
CAPITAL BUDGETING MODEL

by

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WP 1173-80

November 1980

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The purpose of this paper is to offer a suggestion for an improvement in the information content of the simple capital budgeting model via the treatment in the model of cash inflows as a random variable rather than as a single-point estimate.

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One of the major shortcomings or inadequacies of the simple capital budgeting model introduced in most second year and some first year accounting courses as a long-term decision analysis tool is its failure to deal with conditions of uncertainty. This is, I feel, partially due to the inclusion of inappropriate variables in the model.

Unlike the approach employed in the simple, one period break-even model, where the analysis is geared to reveal the impact on profits of changes in the level of sales volume, the simple multi-period capital budgeting-investment analysis analogue relates changes in net present value to changes in the discount rate for a given cash flow pattern. Why it is that the volatility of the most important and volatile element of the analysis, namely sales volume or, more generally, cash flow, is traditionally overlooked remains a mystery.

In order to illustrate the vagaries of the traditional approach I will introduce a simple example which shall also serve the purpose for detailing what I consider a far more significant and relevant approach to the decision analysis.

X. Co. Ltd.

Proposal is to produce and sell "gizmos" at \$1 each. The company is faced with a choice between 2 different machines, A and B, to acquire for purposes of production:-

	A	B
Cost of Machine	\$20,000	\$7,500
Variable cost to make and sell product	10¢	70¢
Contribution margin per unit	90¢	30¢
Annual fixed costs (all cash)	\$10,000	\$1,000
Expected Life	3yrs	3yrs
Estimated Salvage Value	\$0	\$0

Market reserach surveys suggest 25,000 units will be sold annually. The company employs a 10% discount rate in its capital project evaluation.

The traditional investment analysis would proceed as follows:-

	A	B
Expected Revenue (annual)	\$25,000	\$25,000
Variable Costs	<u>2,500</u>	<u>17,500</u>
Contribution Margin	22,500	7,500
Annual Fixed Costs	<u>10,000</u>	<u>1,000</u>
Net Annual Cash Flow	12,500	6,500
X Annuity factor at 10% for 3 years	<u>2.4868</u>	<u>2.4868</u>
Present Value of Cash Flow	31,085	16,164
Purchase Price	<u>20,000</u>	<u>7,500</u>
<u>Net Present Value</u>	\$11,085	\$ 8,664

On the basis of net present value, at a discount rate of 10%, machine A is clearly the preferred choice. Further analysis reveals that at either 8% or 12% machine A is still clearly preferred. In fact no preference for machine B will be revealed by the use of any discount rate less than 20%. The analysis there ends and the choice is made. At various discount rates machine A is always the indicated choice. But what of various sales levels? What would be the preferred choice if expected sales were only 20,000 units annually? Or 30,000? In the event that the firm is introducing a new product onto the market the sales estimate is particularly shaky (of course in such a situation so too would be the estimate of a sales price of \$1 per unit).

It seems vastly more relevant in this instance to consider the impact on net present value (N) of varying sales level(s) rather than varying discount rates. The analysis I propose holds the discount rate fixed at 10% and varies sales, rather than holding sales fixed at 25,000 units and computing net present value at various discount rates. It might be suggested that the sales estimate and the discount rate chosen for evaluative purpose are both very subjective in nature. With this no one can argue. But surely management would be as concerned with the impact of varying sales levels as with different discount rates.

The analysis is best performed graphically and in Figure 1 sales are plotted on the abscissa and net present value on the

ordinate. The mathematics of the analysis are simple for, as can be seen, net present value is a linear function of sales level. The analysis performed above is presented in graphical form and it can be clearly seen that at a sales level of 25,000 units per annum net present value for machine A exceeds that for machine B. But Figure 1 reveals a host of additional information relevant to the choice.

Notice, first, that the sales level has only to fall below 23,378 units annually for machine B to be preferred by the net present value criterion. The sales estimate has to be only 7% too high in the traditional analysis for the wrong choice to be made. Not the slightest clue to this fact was revealed in the earlier analysis however. Second, the chart reveals the fact that "break-even" (zero net present value) for machine B is only 13,386 units while for machine A it is 20,047; a fact which management might be impressed with if the sales level was utterly uncertain.

It seems that, in some sense, machine A is a far riskier proposition than machine B. Notice that at a sales level of 23,378 units the "cross-over point" occurs. That is, if expected sales were 23,378, net present value for each alternative is the same and our traditional analysis would suggest a state of indifference between the two. But for sales levels above and below the cross-over point we see that net present value for machine A changes by a proportion greater than does that for

machine B. That is, for sales below cross-over we are far worse off with machine A and for sales levels above cross-over, far better off. This condition of higher sensitivity of net present value for machine A I shall refer to as risk. This can be quantified quite handily by the slope of the curves in the chart. For machine A a marginal unit sold produces an incremental net present value of \$2.24 while for machine B 75¢ is produced.¹ We might define these as "risk coefficients" and, other things being equal, management might conceivably be indifferent between two projects with different risk coefficients, thus displaying a risk indifferent nature. More likely, however, would be the choice of the alternative with the smaller risk coefficient, displaying a more typical state of risk aversion on the part of management. To incorporate such a factor as the risk averse (or otherwise) nature of management in the analysis is a far cry from our traditional uncomplicated analysis. But the relevance of such a factor cannot be denied.

How does our decision problem look now? Let us recap. Machine A is preferred on the basis of its expected net present value, or what I shall refer to being μ - NPV preferred to B. Machine B is, however preferred in a "break-even" sense (B/E preferred) and also (assuming management is risk-averse) in a risk sense (risk preferred). How is our decision to be made?

It seems that we must attach a probabilistic dimension² to the analysis in order to derive any rigorous decision. We will

recall that sales only had to fall below their expected amount (μ -sales) of 25,000 units annually to 23,378 units in order that machine B be preferred in terms of net present value. Perhaps the probability of such an outcome is very low, that is the distribution of sales is very highly concentrated around μ (25,000). Alternatively, the probability may be quite high in the case of marked dispersion of sales about μ . We would like to have some indication of the expected variation of sales levels about μ , or statistically, we would like to know the standard deviation of sales as well as their expected value of 25,000 units, which we already know. This introduces difficulties into the analysis for, without knowledge of the distributional form of sales (i.e. normal, poisson, gamma, etc.) we generally cannot discover its standard deviation. For our purposes I introduce the greatly simplifying assumption that sales are normally distributed with μ of 25,000.³ This provides for computational ease and also allows a convenient, albeit crude, opportunity to elicit an estimate of the standard deviation. The sales manager or market research team responsible for the estimate of $\mu=25,000$ are once again consulted and asked "within what range about 25,000 units annually, are you $2/3$ (i.e. $66\frac{2}{3}\%$) certain that sales will fall?" This provides an estimated of the standard deviation of sales (σ).⁴ We will recall that approximately 68% (68.26% to be exact) of the area under the normal curve lies within the range $\mu \pm 1\sigma$.

Suppose that σ is thus estimated to be 2,000 units. Some vital information related to the decision is now available. Management, perhaps concerned about the disquieting fact that sales only have to fall to 23,378 in order for machine B to be more desirable than machine A, are now possibly delighted to learn that the probability of their doing so is only approximately 0.21. The probability of a sales level inadequate to achieve "break-even" is, in the meantime, approximately zero. Management is thus in a position to virtually ignore the undesirable qualities of machine A with respect to "break-even" and with machine A offering a higher expected net present value than B with probability 0.89 will probably choose machine A.

By contrast, the sales estimate of 25,000 units may have been far less certain, the distribution displaying a standard deviation of, say, 5,000 units. In such a case, the probability of failing to achieve "cross-over" level sales jumps to 0.37 and failure to achieve break-even for machine A increases to 0.16. The decision in this case is not so clear and management displaying sufficiently risk averse characteristics might conceivably elect to choose machine B inspite of the fact that it offers only a 37% chance of producing a new present value higher than machine A. Clearly, as this percentage nears 50%, management is more likely to choose machine B. Of course the percentage cannot reach 50% in this case of a symmetric distribution since μ -sales exceed cross-over sales.

A handy visual aid in this analysis is to superimpose the probability distribution on the graph presented in Figure 1. The resulting presentation appears in Figure 2. By blocking out the area shown, attention can be focused on only the probabilistically "relevant range" of sales volume that is from 19,000 to 31,000 units annually ($\mu \pm 3\sigma$).

The model I have presented is only marginally more complex than the simple capital budgeting analysis. But I believe that the added information made available by the model is of significant value. Three added assumptions and one additional item of information are necessary over and above the information and assumptions of the simple model. These are worthy of summarization. First, it is necessary to assume that the chosen discount rate is fairly objectively determinable. Textbook expositions of the means of establishing a discount rate traditionally suggest that the choice can be highly subjective. However more recent literature in the finance area is beginning to discredit the traditional text-book explanation.⁵ Secondly, in order to develop any useful probability statements with any ease, it is necessary to assume that sales are normally distributed. As was mentioned, more appropriate distributional forms exist but the potential refinements involved in a more accurate model are empirical issues. Finally, and perhaps most importantly, the model is workable, in its present form, only where expected sales volume is assumed to be equal for each year

of the life of the project. Equal service life and zero salvage value were assumed in the model only to simplify the analysis. These can be incorporated without much difficulty.

The only added informational need of the model is an estimate of the standard deviation of the sales distribution. A suggestion for a simple (albeit crude) means of establishing this was made.

In conclusion it should be said that as in the case of many simple decision models popularized in management accounting and business finance, they are simple for one good reason: added sophistication is not worthwhile in many cases. I have I feel introduced an example of a decision problem for which, however, the simplifying assumptions of the traditional model render it very incomplete and inadequate. Moreover, the added usefulness of the proposed approach is provided at minimal cost.

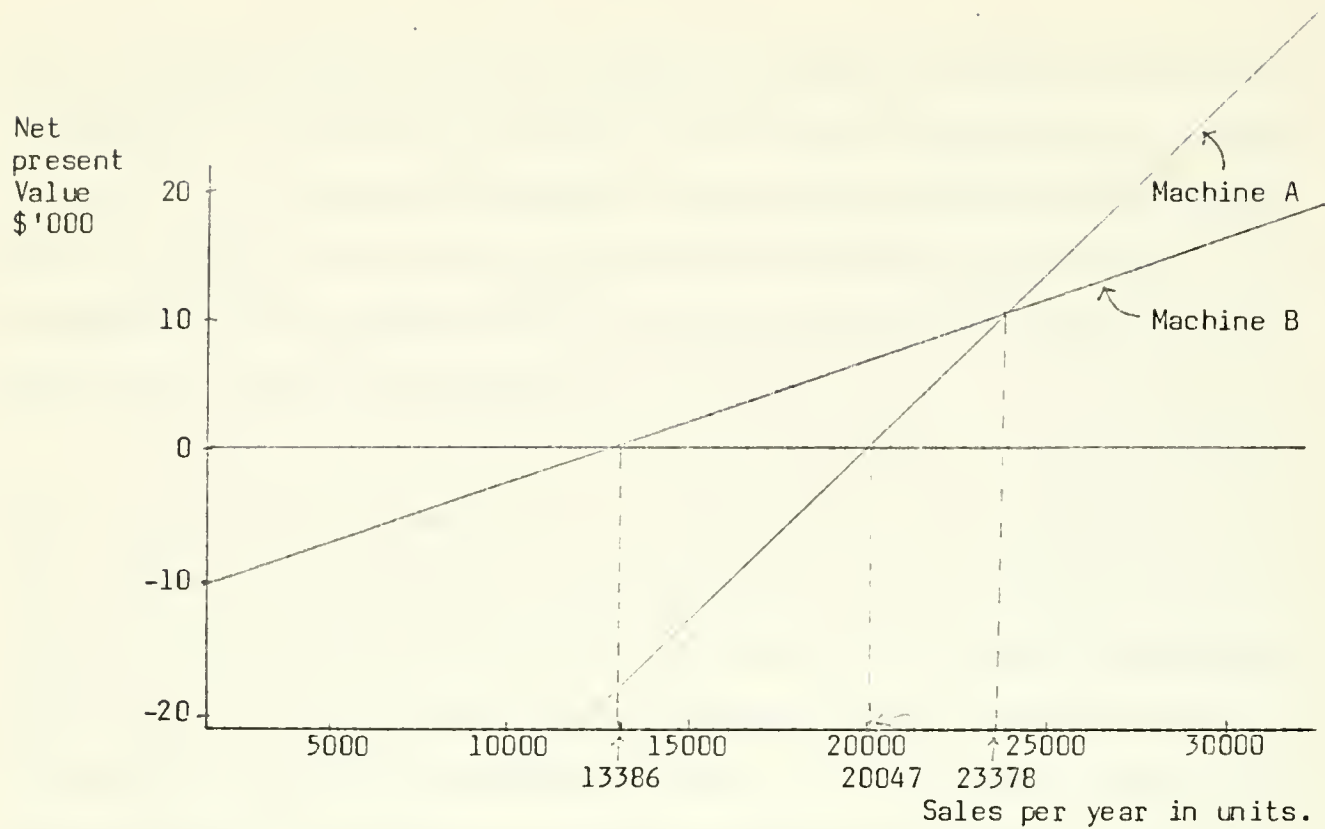


FIGURE 1

Net Present Value as a function of Product Sales

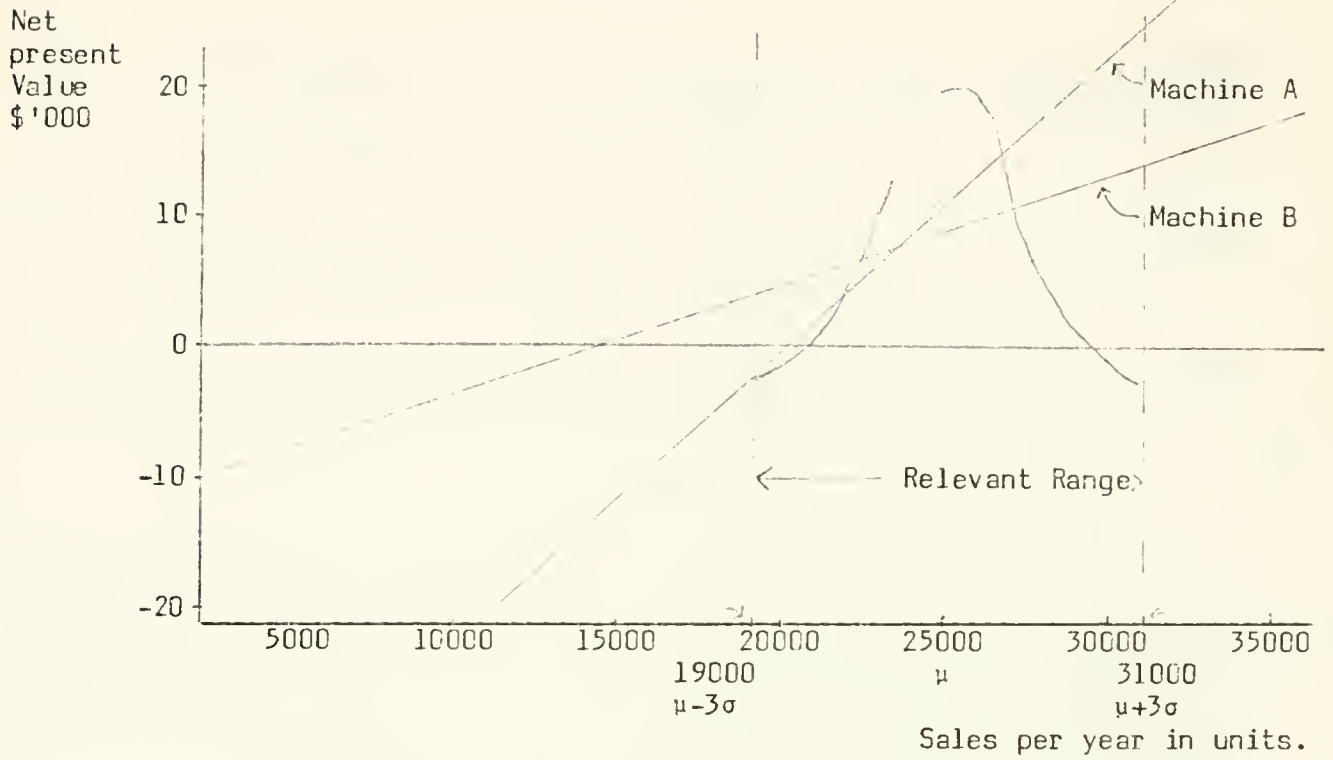


FIGURE 2

The "Relevant Range" Depicted

FOOTNOTES

¹Mathematically, the partial derivative of each machine's net present value function produces the above results:-

Machine A:-

$$N_1 = d (.9s - f_1) - p_1 \cdot \frac{\partial N_1}{\partial s} = .9d = .9 \times 2.4868 = \$2.24$$

Machine B:-

$$N_2 = d (.3s - f_2) - p_2 \cdot \frac{\partial N_2}{\partial s} = .3d = .3 \times 2.4868 = \$0.75$$

where N is net present value

d is the annuity factor for the 10% discount rate i.e.
2.4868

s is the sales level

f is the annual fixed cost

p is purchase price

²The accommodation of probability distributions for the variables involved in a capital budgeting model is not a novel idea. See for example D. B. Hertz ("Risk Analysis in Capital Investment", in Harvard Business Review Vol. 42, No. 1, Jan-Feb, 1964, pp. 95-106) who outlined a procedure for attaching probabilistic dimensions to no less than nine relevant variables in generating a single probability distribution for rate of return.

³It has been empirically demonstrated that, in fact, the log-normal distribution of sales is an accurate approximation. See Holt, C. C., Modigliani, F., Muth, J. F., and Simon, H. A., "Planning Production, Inventories and Work Force", (Prentice-Hall 1960) p. 283.

⁴This idea is borrowed from Jaedicke, R. K., and Robichek, A. A., "Cost-Volume-Profit Analysis Under Conditions of Uncertainty", The Accounting Review, October, 1964 pp. 917-926.

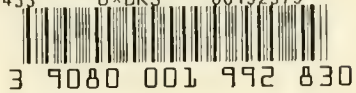
⁵See, for example, Rubinstein, M., "A Mean-Variance Synthesis of Corporate Financial Theory", Journal of Finance, March 1973.

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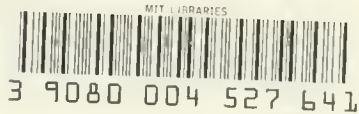
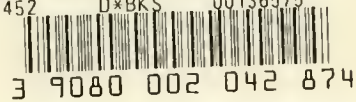
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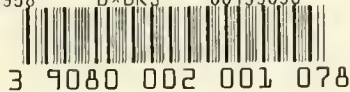


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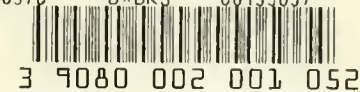


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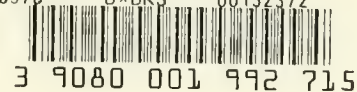
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